APPLYING PETRI NETS EXTENSIONS TO MODELING COMMERCIAL BANK ACTIVITY

Igor ENICOV[1], PhD, Professor,
Free International University of Moldova

The relevance of the study is determined by the need to improve the methods of modeling and simulating commercial bank activity, including for the purpose of calculating, controlling and managing the risk of the bank, in the context of the transition to the application of Basel III standards. This improvement becomes necessary due to a direct transition to new regulatory standards when the internal assessments of the main risks become the initial data for calculating the capital adequacy of a bank. The purpose of this article is to argue the opportunity to formulate a theory of the commercial bank model on the extensions of Petri nets theory. The main methods of research were the method of scientific abstraction and method of logical analysis. The main result obtained in the study and presented in the article is the argumentation of the possibility to analyze the quantitative and qualitative characteristics of a commercial bank with the help of Petri net extensions.

Keywords: *commercial bank, modeling, bank risk, Petri nets, extensions of Petri nets theory, qualitative properties, quantitative properties.*

Actualitatea studiului este determinată de necesitatea perfecționării metodelor de modelare și simulare a activității băncii comerciale, inclusiv în scopul evaluării, controlului și gestiunii riscului băncii, în contextul trecerii la aplicarea standardelor Basel III. Această perfecționare se impune odată cu trecerea efectivă la cadrul nou de reglementare când estimările interne ale riscurilor de bază devin date primare pentru calculul cerinței de capital al unei bănci. Scopul prezentului studiu este de a argumenta oportunitatea formulării unei teorii a modelului băncii comercale pe extensii ale teoriei rețelelor Petri. Metodele principale de cercetare au fost metoda abstracției științifice și metoda logică. Principalul rezultat obținut în articol, urmare a cercetării, a fost argumentarea posibilității explorării proprietăților calitative și cantitative ale activității băncii comerciale pe extensii ale rețelelor Petri.

Cuvinte-cheie: *bancă comercială, modelare, riscul băncii, rețele Petri, extinderi ale teoriei rețelelor Petri, proprietăți calitative, proprietăți cantitative.*

Актуальность исследования определяется необходимостью улучшения методов моделирования и симуляции деятельности коммерческого банка, в том числе с целью расчета, контроля и управления риском банка, в контексте перехода к применению стандартов Базель III. Это улучшения становится необходимым так как при непосредственном переходе на новые стандарты регулирования внутренние оценки основных рисков становятся исходными данными для расчёта достаточности капитала банка. Основными методами исследования были метод научной абстракции и метод логического анализа. Основным результатом полученным в статье, в результате исследования, является аргументирование возможности исследования количественных и качественных характеристик коммерческого банка с помощью расширений сетей Петри.

Ключевые слова: *коммерческий банк, моделирование, банковский риск, сети Петри, расширения теории сетей Петри, качественные характеристики, количественные характеристики.*

JEL Classification: G17, G21, G31, C60.
UDC: 336.713:330.4

Introduction. In recent years, the National Bank of Moldova has carried out some remedial measures to improve the situation in the banking sector. Thus „since June 2015, the National Bank of Moldova has been benefiting from the assistance provided by the central banks of Romania and the

[1]© Igor ENICOV, enicov.igor@gmail.com

Netherlands through a Twinning project related to strengthening the NBM capacity in the field of banking regulation and supervision in the context of the Directive 2013/36 / EU and Regulation 575 / 2013 related to the banking sector (so-called CRD IV package)" [9].

Also, „entry into force of the Law on banking activity and activity of investment firms, which is planned for the end of 2017 and the subsequent entry into force of the secondary normative framework for implementing its provisions will address the transition from international standards Basel I to latest standards in the field – Basel III" [8].

Even if we assume that "the NBM will not allow the use of the model based on the fundamental internal ratings-based approach (FIRB) and the model based on the advanced internal ratings-based approach (AIRB) for a long period of time" [9], we believe that commercial banks are interested in developing existing practices of modeling and simulating activities, including risk management purposes. In other words, we must improve the formalization of logical and mathematical representation of connections, regularities and the explanatory theory for the operations in commercial bank. The main purpose of this article is to justify the opportunity to formulate an explanatory theory on the extensions of Petri nets theory.

We believe that attaining similitude between commercial bank and its model by Petri nets has a number of advantages compared to other instruments. This involves some opportunities of continuous improvement of the theoretical model, the possibility of slight changes of the degree of homomorphism, the possibility of concomitantly including variables typical of deterministic models, stochastic models, as well as fuzzy (vague) models. It is important that Petri nets have a graphic representation with a particularly effective impact on intuitively understanding the systems dynamics.

Basic content. It is known that "Petri nets can model phenomena characteristic of discrete event systems such as succession (one evolution follow another), choice or conflict (selecting one of several evolution possibilities), concurrency (state of parallel evolutions), synchronization (completion of parallel evolutions), mutual exclusion (mutual determination of some evolutions), which can be formulated in timed or non-timed contexts. In addition, the study of Petri nets is usually accompanied by a visual approach through expressive charts" [6].

It is known that a Petri net is a representation of a system that can decompose into a structural part and a dynamic one [5].

A structural part of a Petri net is essentially a bipartite directed graph (P, T, A, Kp, w), where:

$P = \{p_1, p_2, \ldots, p_n\}$ is a finite set of *places;*

$T = \{t_1, t_2, \ldots, t_k\}$. is a finite set of *transitions* $P \cap T = \varnothing$;

A is a set of arcs, a subset of the set $(P \times T) \cup (T \times P)$;

$K_p : P \to IN \cup \infty$ is a function of the place capacity (the ability of a place is considered implicitly unlimited);

w is a function that assigns weight to arcs, $w: A \to \{1,2,3 \ldots\}$ (the weight of an arc is considered implicitly uniform).

The dynamic component of a Petri net, initially marked (P, T, A, w, M_0) consists in identifying ways (laws) of the evolution of these markings.

Un The marking of a Petri net at a certain moment of its evolution is defined by a vector $M=[m_1, m_2,\ldots,m_n]$, where n is the number of places in a Petri net, and $m_i = M(p_i)$, with index i showing the number of tokens in a place p_i, $M(p_i) \in \{0, 1, 2, \ldots\}$. M_0 is the initial marking of a Petri net.

The state of a certain Petri net (structurally defined) is completely described by its marking $M = [m_1, m_2,\ldots, m_n]$. The state space of a marked Petri net is fully defined by the reachable marking out of M_0, that is to say, all n-dimensional vectors, whose elements are positive $M = \{0, 1, 2, \ldots\}^n$.

In order to be able to use a Petri net for modeling a discrete event system, it must be equipped with a dynamic and defining mechanism similar to the one of the state transition. This mechanism is highly suggestive in its representation of the tokens exchange between the net places determined by enabling and firing of the net transition. A transition $t_j \in T$ in a marked Petri net is enabled if:

$M(p_i) >= w(p_i, t_j)$, for each $p_i \in I(t_j)$; $M(p_k) <= K_p(p_i)-w(t_j, p_k)$, for each $p_k \in O(t_j)- I(t_j)$;

$M(p) <= K_p(p)-w(t_j, p) + w(p, t_j)$, for each $p \in O(t_j) \cup I(t_j)$.

In other words, in order to produce a transition, first it is required that the number of the tokens on a place at the input of the transition is, at least, the same as the weight of the arcs that connect the corresponding places of the transition. When a transition is enabled by current marking M, (notation

$M\ [t_j >)$, we say that this transition can be fired. The set of the enabled transitions by current marking M is noted as $T(M)$.

The firing of a transition is visually equivalent to withdrawing a number of tokens (equal to the weight of the connecting arc) on each place of the transition input and adding a number of tokens (equal to the weight of the connecting arc) on each place of the transition output:

$M'(p_i) = M(p_i) - w(p_i, t_j)$, for each $p_i \in I(t_j) - O(t_j)$;

$M'(p_k) = M(p_k) + w(t_j, p_k)$, for each $p_k \in O(t_j) - I(t_j)$;

$M(p) = M(p) - w(p, t_j) + w(t_j, p)$, for each $p \in O(t_j) \cap I(t_j)$.

The function of the state transition of a marked Petri net, , $f: \{0, 1, 2, \ldots\}^n \times T \rightarrow \{0, 1, 2, \ldots\}^n$ is defined for transition $t_j \in T$, if it is enabled, i.e. if $M(p_i) \geq w(p_i, t_j)$ for each p_i out of $I(t_j)$.

According to transition $f(x,t_j)$, the new marking is:

$M' = f(x, t_j)$, where $M'(p_i) = M(p_i) - w(p_i, t_j) + w(t_j, p_i)$, $i = 1, 2, 3, \ldots$

This relation is noted as $M[t_j > M'$. We say that M' is directly reachable from M by firing transition t_j.

The analysis of the marked Petri nets can be performed on the basis of their dynamic behavior or their structure.

The first method is called the analysis of reachability because its main goal is to identify the set of states (reachable). It uses algebraic rules which describe the processes of enabling and firing transitions and lead to representing the dynamic evolution of a Petri net through some equations.

The second method, which is a structural analysis, focuses on eliminating space derivation resulting in avoiding the "state explosion" problem.

Petri nets extensions correspond to the models that have acquired some additional rules to allow treating a greater number of applications and models. The most significant models that extend the descriptive power of a model are generalized Petri nets, timed Petri nets, stochastic Petri nets, continuous Petri nets and hybrid Petri nets.

Generalized Petri nets adequately describe the discreet events processes, which take place in a system [3]. In generalized Petri nets, events are viewed as transitions. A transition, or an event, takes place only when it is enabled, which means that a set of conditions is applied. Places can be input transitions in cases when they are associated with the conditions, which must be applied in order to enable it. Or, they can be output transitions and in this case they represent new conditions which appeared due to the production of the transition. It is important that generalized Petri nets involve local determination of the state change, thus the transitions, i.e. events, can be controlled independently [5]. In other words, in case of modeling commercial bank activity, we can have a Petri net in which the transition representing an event has an input place, which stands for the state of the bank with a range of conditions to be fulfilled so that the event takes place, and an output place, which stands for a new condition that appeared after producing the event.

In case of Petri nets, the net dynamics are associated with a succession of transitions that are dependent only on logical considerations. As it was mentioned above, transitions can be fired only on condition that they are enabled, i.e. only if a set of described conditions is carried out in the input place. In this context, time is not taken into account, i.e. we do not relate the production of the event to the necessary tie intervals. Yet, in order to simulate commercial bank activity in some cases it is necessary to consider time. In these cases, the necessary time can be related to either place or transition.

In case it is necessary to take into account time in a modeled system, then it can be related to places and transitions in the following way:

a) The time interval of the transition is the time between the start (the consumption of tokens at the input place) and the completion of the event (the production of tokens at the output place). These time intervals are called action time.

b) The time interval of places is the time needed for consumption so that tokens can become permanent in a place until the moment they are able to contribute to the firing of the following transition. These time intervals are called waiting times [3].

A variant of generalized timed Petri nets, which can be used to model and assess the risks of commercial bank activity, can be observed in [3].

In the context of modeling commercial bank activity, action times can be represented by, for example, the time required for making a bank transfer (in this case, the transition is the time consumed for making the bank transfer). Similarly, the waiting times can be time intervals necessary for firing a transition, i.e. the commencement of the transfer, or the termination of the transfer. Starting with the initiation of the transfer, the time is consumed to verify the conditions necessary for the transfer, action times are the time needed to verify each of the existing conditions to debit the payer's account. When the conditions are fulfilled, in other words, when the transition is enabled (the event – the transfer is fired), there will be consumed the waiting times, i.e. the durations required to verify the necessary conditions to enroll the money to the beneficiary's account.

Timed Continuous Petri nets extensions were first introduced by H. Alla and R. David in [1]. The particularities of this Petri nets extension are in the fact that the marking of a place, the number of the tokens and the incidence function are defined by a real number. In the extension of this kind, firing of the transition is carried out by a continuous flow of fluid. Timed continuous Petri nets allow the modeling of situations in which the number of reachable markings becomes too high to be modeled by Petri nets [4].

For modeling real processes, transitions of generalized Petri nets can have durations related to random or concrete values. In case of random values, we obtain a generalized stochastic Petri net. In other words, in this case, the duration needed to produce an event will follow a probability distribution that will relate to the corresponding duration. In case of concrete values, we can observe a generalized deterministic Petri net, which in its turn is a specific case of generalized stochastic Petri nets.

In the context of a stochastic Petri net, if the transition is enabled, there will be selected all the markings necessary for the production of the respective transition and they will be relevant for a time interval generated according to the law of repartition associated with the respective transition. If it is possible to produce more transitions simultaneously, first there will be carried out the transitions that have the shortest delay time.

Any generalized stochastic Petri net with exponential distributions, a Markovian Petri net, can be assigned to a homogeneous continuous Markovian chain. So, along with the general methods used to analyze generalized Petri nets, there can be used the methods of analyzing a homogeneous Markovian chain [3].

The choice between the various possible interpretations of Markovian Petri nets generally depends on the modeling techniques and objectives. According to [2], a Markovian Petri net is a generalized Petri net in which:

each transition t_k is associated with a random variable ξ_k that models the duration needed for action t_k while firing;

every random variable ξ_k is characterized by a negative-exponential distribution function (with a corresponding parameter) of the cumulative time of firing of each transition apart.

Based on the above, we can determine a set of possible executions of a Markovian Petri net, which is a Markovian random process with continuous, discrete states. Due to lack of memory of negative exponential law, at each iteration, "memory loss of cumulative actions" occurs; thus the behavior of a Markovian Petri net is described by a continuous Markovian chain, i.e. the states previously held have no influence on the subsequent evolution. General interpretation of the behavior of a Markovian Petri net allows to create a simple model of discrete event systems.

Timed hybrid Petri nets are well suited for modeling continuous operation with naturally continuous flows. However, in a banking system operation, processes are typically discrete-continuous and there can often be situations of malfunctions when one or more resources are not available. The concurrency of these events leads to a sudden change in the operating mode of the net, which is equivalent to having another timed Petri net unexpectedly.

Suchlike situations can be modeled through generalized hybrid Petri nets, which contain continuous places and transitions and discreet places and transitions. Put in other words, in a hybrid Petri net a set of places and transitions is divided into two distinct parts: every place and every transition can be either discreet or continuous, in which the marking of the continuous place is a real number, whereas the marking of a discreet place is an integer [5].

Results and conclusions. Generalizing all the above mentioned points, we can affirm that the construction of a model of commercial bank activity can be made by identifying two discrete sets (finite or infinite): T-state space and E-event set as well as formulating a mathematical description of the connections by means of which the occurrence of events from set E determines transactions in state space T. This can be achieved:

a) qualitatively, when logical behavior is considered, regardless time, which also means that the mathematical model does not contain information on the moments when events are produced and in this case we speak of a logical, untimed model;

b) and/or quantitatively, when time-dependent behavior is considered, which means that in the mathematical model events are presented as pairs *(e1,m1), (e2,m2), ..., (en,mn)*, where *m1, m2, ..., mn* note the moments of occurrence of the respective events and in this case we speak of a timed model. Thus, such models reveal the quantitative properties of systems behavior. It is important that the process of identifying moments can be deterministic (deterministic models) or probabilistic (stochastic models).

Such quantitative and qualitative properties of commercial bank activity can be explored through Petri nets extensions, ensuring a relative simplicity of formulating logical and mathematical representation of connections, regularities and explanatory theory for processes of commercial bank and, simultaneously, providing graphic support with a highly efficient impact on understanding the dynamics of the model intuitively.

REFERENCES

1. HASSANE, Alla, RENE, David. Discrete, Continuous and hybrid Petri nets. In: Journal of Systems Circuits and Computers. 1998, no. 8 (1), pp. 159-188.
2. AJMONE-MARSAM, M. et al. A class of Generalized Stochastic Petri Nets for Performance Evaluation of Multiprocessor System. In: ACM Transactions on Computer Systems. 1984, vol. 2, may, pp. 93-122.
3. GUŢULEAC, E. *Evaluarea performanţelor sistemelor de calcul prin reţele Petri stocastice*. Chişinău: Tehnica Info, 2004. 276 p. ISBN 9975-63-228-9.
4. ENICOV, I., GUŢULEAC, E. Modelarea activităţii băncii comerciale prin reţele Petri hibride temporizate. In: Studia Universitatis. Ştiinţe exacte şi economice. Chişinău 2007, anul I, nr. 2, pp.218-225.
5. ENICOV, I., GUŢULEAC, E. Modelarea şi evaluarea performanţelor băncii comerciale prin reţele Petri. In: Studia Universitatis. Ştiinţe exacte şi economice. Chişinău, 2007, anul I, nr. 2, pp. 177-178.
6. PĂSTRĂVANU, O., MATCOVSCHI, M., MAHULEA, C. *Aplicaţii ale reţelelor Petri în studierea sistemelor cu evenimente discrete*. Iaşi: Ed. Gheorghe Asachi, 2002. ISBN 973-8292-86-7.
7. BEDA, O. et al. Problems of financial stability of commercial banks in the Republic of Moldova. In: Economie şi Sociologie = Economy and Sociology. 2015, nr. 4, pp. 49-54.
8. Situaţia financiară a sectorului bancar în anul 2016 [accesat 12 februarie 2017]. Disponibil: http://bnm.md/ro/content/situatia-financiara-sectorului-bancar-anul-2016
9. Strategia de implementare a standardelor BASEL III prin prisma cadrului legislativ european (CRD IV) [accesat 17 ianuarie 2017] Disponibil: http://bnm.md/files/Strategia%20(web).pdf

Recommended for publication: 31.03.2017